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Rearrangement of the Fermi Surface of Dense Neutron Matter and Direct Urca Cooling of Neutron Stars

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Abstract

It is proposed that a rearrangement of single-particle degrees of freedom may occur in a portion of the quantum fluid interior of a neutron star. Such a rearrangement is associated with the pronounced softening of the spin-isospin collective mode which, under increasing density, leads to pion condensation. Arguments and estimates based on fundamental relations of many-body theory show that one realization of this phenomenon could produce very rapid cooling of the star via a direct nucleon Urca process displaying a T^5 dependence on temperature.

1. INTRODUCTION

The EINSTEIN, EXOSAT, and ROSAT orbiting X-ray observatories have measured surface temperatures of certain neutron stars and set upper limits on surface temperatures of others ([3]). The data for the supernova remnants in 3C58, the Crab, and RCW103 indicate relatively slow cooling, while that for Vela, PSR2334+61, PSR0656+14, and Geminga point to substantially more rapid cooling. In the so-called standard scenario for neutron-star cooling, the primary role is played by the modified Urca process ($nn \to npe^-\bar{\nu}_e$; $npe^- \to nn\nu_e$), first considered by Bahcall & Wolf (1965) and later reexamined by Friman & Maxwell (1979) in terms of an in-vacuum one-pion exchange model. Cooling simulations based on the results of these works play out the slow scenario of thermal evolution and fail to explain the rapid cooling of some stars. This picture is profoundly altered when in-medium effects are taken into account. Modification of NN, πN, and KN interactions with increasing density may be so strong that pion ([11, 12, 2]) and kaon ([Brown 1994]) condensates form in the interior region of a high-mass neutron star, leading to a dramatic increase of the neutrino luminosity ([12, 10, 16]).

Here we shall demonstrate that softening of the spin-isospin (pion) collective mode in dense neutron matter as predicted by Migdal (1978) could give rise to a rearrangement of single-particle degrees of freedom prior to the onset of pion condensation and open a new channel of neutrino cooling of neutron stars, by giving access to the *direct* Urca process. This process does not involve a neutron (or other) spectator and, if allowed, is an extremely efficient cooling mechanism ([15]).

2. BUBBLE REARRANGEMENT OF THE FERMI SPHERE

A rearrangement of single-particle degrees of freedom takes place if the necessary condition for stability of the normal state of a Fermi liquid is violated. At $T = 0$ this condition requires that the change of the ground-state energy E_0 remain positive for any admissible variation $\delta n(p)$ of the Landau quasiparticle distribution $n(p)$ away from the normal-state step-function distribution $\theta(p - p_F)$. Formally,

$$\delta E_0 = \int \xi\left(p, n(p)\right) \delta n(p) \frac{\mathrm{d}^3 p}{(2\pi)^3} > 0\,, \tag{1}$$

where $\xi(p, n(p)) \equiv \varepsilon\left(p, n(p)\right) - \mu$ is the energy of a quasiparticle relative to the chemical potential μ. The condition (1) fails if a depression with $\xi < 0$ forms in the spectrum $\xi(p)$ at $p > p_F$; it likewise fails if there arises an elevation with $\xi > 0$ at $p < p_F$. The rearrangement is precipitated when the density ρ reaches a critical value ρ_{cF} at which there emerges a bifurcation and a new root $p = p_0$ of the relation

$$\xi\left(p, n(p); \rho_{cF}\right) = 0\,, \tag{2}$$

which ordinarily serves merely to specify the Fermi momentum p_F.

The simplest kind of rearrangement of the momentum distribution $n(p)$ of quasiparticles of given spin projection retains the property that its values are restricted to 0 and 1, but the Fermi sea becomes doubly connected ([5]). At densities exceeding the critical value ρ_{cF}, the normal-state distribution $\theta(p - p_F)$ is modified by the presence of a "bubble," or absence of particles, over the range $p_i < p < p_f < p_F$, with the inner surface p_i located relatively close to the origin and the Fermi momentum p_F readjusted to maintain the prescribed neutron density. The distance $p_f - p_i$ between the two new Fermi surfaces lying interior to p_F can be estimated using the formula $\xi(p \to p_0, n(p); \rho) = \xi_0(\rho - \rho_{cF}) - A(p - p_0)^2$, where A and ξ_0 are positive constants. This formula embodies the essential properties that $\xi(p)$ is negative for any $p < p_F$ at $\rho < \rho_{cF}$ and that its maximum value first reaches zero at $\rho = \rho_{cF}$. Employing this parametrization in the relation (2), it is found that no bifurcation point exists for $\rho < \rho_{cF}$, whereas two solutions arise for $\rho > \rho_{cF}$, with the distance between the two new Fermi surfaces growing in proportion to $\sqrt{\rho - \rho_{cF}}$.

Were such a rearrangement to occur in the neutron subsystem of neutron-star matter, the emergence of new Fermi surfaces situated at lower momenta would permit the direct nucleon Urca process to operate at a much lower density than hitherto considered possible ([15, 9]). In this process, the tandem reactions $n \to pe^-\bar{\nu}_e$ and $pe^- \to n\nu_e$ are driven by thermal excitations. The condition of high degeneracy prevailing even in young neutron stars implies that these excitations remain close to the Fermi surfaces of the participants. For the direct Urca mechanism to contribute appreciably to neutron-star cooling, momentum conservation then demands satisfaction of the triangle inequalities $|p_{Fp} - p_{Fe}| \leq p_{Fn} \leq p_{Fp} + p_{Fe}$ among the proton, electron, and neutron Fermi momenta. With the conventional singly-connected neutron Fermi sphere, these conditions are met only at very high baryon densities where the proton Fermi momentum k_{Fp} reaches sufficiently large values. The best current estimates yield a threshold baryon density of at least five times the saturation density ρ_0 of symmetrical nuclear matter ([2]). On the other hand, if the neutron subsystem undergoes a rearrangement that allows for thermal excitations of neutron quasiparticles at much lower momenta than p_{Fn}, specifically at p_i and p_f in the bubble rearrangement scenario, the triangle inequalities are much more easily satisfied and the direct Urca process is greatly facilitated.

We next provide a quantitative basis for this qualitative idea, by appealing to established methods of microscopic many-body theory. For a bifurcation point to arise in the solution of equation (2), both the spectrum $\xi(p)$ and the scalar component $f(\mathbf{p}_1, \mathbf{p}_2; \mathbf{k} = 0)$ of the Landau amplitude of the quasiparticle interaction must depend strongly on momentum. The connection ([1], p. 37)

$$\frac{\partial \xi_(p)}{\partial \mathbf{p}} = \frac{\partial \xi^0(p)}{\partial \mathbf{p}} + \frac{1}{2}\mathrm{Tr}_\sigma \mathrm{Tr}_{\sigma_1} \int f(\mathbf{p}, \sigma, \mathbf{p}_1; \sigma_1, \mathbf{k} = 0) \frac{\partial n(p_1)}{\partial \mathbf{p}_1} \frac{\mathrm{d}^3 p_1}{(2\pi)^3} \tag{3}$$

between these two quantities, wherein $\xi_0(p)$ is the free spectrum and spin dependence is made explicit, is usually employed to find the effective mass M^* in terms of the first harmonic of the expansion of the amplitude $f(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2; 0)$ in Legendre polynomials. We exploit this relation in the search for new roots of equation (2).

Within the integral in (3), the quasiparticle interaction amplitude $f(\mathbf{p}_1, \mathbf{p}_2; \mathbf{k} = 0)$ may be replaced by $(M^*/M)\Gamma^k(\mathbf{p}_1, \mathbf{p}_2)$, the scattering amplitude $\Gamma^k(\mathbf{p}_1, \mathbf{p}_2) \equiv \Gamma(\mathbf{p}_1, \mathbf{p}_2; \mathbf{k} = 0)$ modified by an effective-mass factor ([1]). The requisite strong momentum dependences can arise if the system approaches a second-order phase transition that occurs at some critical density ρ_c where a collective mode of frequency $\omega_s(k)$ collapses at wave number $k = k_0$ and the corresponding susceptibility diverges along with the scattering amplitude. Near such a soft-mode critical point, the singular part Γ^s of the scattering amplitude $\Gamma(\mathbf{p}_1, \mathbf{p}_2; \mathbf{k})$ at momentum transfer $\mathbf{k}$ may be expressed quite generally by ([6])

$$\frac{M^*}{M}\Gamma^s_{\alpha\kappa;\beta\lambda}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{k}, \rho \to \rho_c) = -O_{\alpha\kappa} O_{\beta\lambda} D(k) + O_{\alpha\lambda} O_{\beta\kappa} D(|\mathbf{p}_1 - \mathbf{p}_2 + \mathbf{k}|)\,, \tag{4}$$

where O denotes the vertex determining the structure of the collective-mode operator (e.g. $O = (\sigma \cdot \mathbf{n})\tau$ for the spin-isospin mode with pion quantum numbers, where $\mathbf{n}$ is a unit vector along the relevant momentum). In deriving (4), antisymmetry of the two-particle wave function under interchange of the coordinates and spins of the two particles has been invoked. The propagator D may be parametrized as $D(k) = \left[\beta^2 + \gamma^2(k^2/k_0^2 - 1)^2\right]^{-1}$, where $\beta(\rho)$ measures the proximity to the phase transition point, with $\beta(\rho \to \rho_c) \sim \rho_c - \rho$ (cf. Dyugaev (1976)).

The essential messages of the preceding development are that the singular part $\Gamma^s(\mathbf{p}_1, \mathbf{p}_2; \mathbf{k} = 0) \sim D(|\mathbf{p}_1 - \mathbf{p}_2|)$ of the scattering amplitude depends on the difference $\mathbf{p}_1 - \mathbf{p}_2$ and that as one approaches the soft-mode phase transition point this dependence becomes quite strong. We assume that the remaining contributions to $\Gamma(\mathbf{p}_1, \mathbf{p}_2; \mathbf{k} = 0)$ can be adequately incorporated by renormalization of the chemical potential μ. Equation (3) is then easily integrated to produce an explicit expression suitable for calculation of the single-particle spectrum,

$$\xi(p) = \xi^0(p) + \frac{1}{2}\mathrm{Tr}(O_{\alpha\lambda}O_{\lambda\alpha}) \int D(|\mathbf{p}-\mathbf{p}_1|)n(p_1)\frac{\mathrm{d}^3p_1}{(2\pi)^3}\,. \tag{5}$$

We now apply this equation to dense neutron matter in the vicinity of the second-order phase transition associated with neutral pion condensation, which is engendered by the softening of the spin-isospin mode having π^0 quantum numbers ([11]). It has been predicted that the collapse of this mode will take place at a neutron density $\rho_c = \rho_{c\pi}$ in the range $(0.2 - 0.5)$ fm^{-3} (roughly 1–3 times ρ_0), depending on theoretical assumptions ([12, 2]). Unfortunately, there is as yet no definitive microscopic treatment of neutron-star matter from which one can extract or derive quantitatively reliable values for the input parameters β, γ, and k_0 of our model.

In this situation, a reasonable strategy is to perform calculations based on expression (5) for several choices of the parameters of the microscopic model. Substituting (5) into relation (2), one finds the critical density ρ_{cF} for the onset of a bifurcation of the latter equation. For $\rho > \rho_{cF}$ this equation then determines two new momenta p_i and p_f where $\xi(p)$ vanishes, which delimit the bubble region of $n(p)$ and between which $\xi(p)$ is positive. Representative numerical results for the spectrum $\xi(p)$ are plotted in Fig. 1. Results for the phase diagram of dense neutron matter in the ρ/ρ_0 versus β^2/m_π^2 plane are displayed in Fig. 2. Different values of γ are considered, while keeping the parameter k_0 fixed at the value $0.9p_{Fn}$ suggested by earlier numerical calculations ([12]).

It is evident that variation of the parameters β, γ, and k_0 within sensible bounds can have strong effects on the phase diagram and therefore on the extent of the phase with rearranged quasiparticle occupation. Nevertheless, our numerical study has documented three salient features of the bubble rearrangement. *First*, the critical density ρ_{cF} for the rearrangement is less than the critical density $\rho_{c\pi}$ for pion condensation. Since both phenomena stem from the strong momentum dependence of the Landau amplitude $f(\mathbf{p}_1, \mathbf{p}_2; \mathbf{k} \to 0)$, rearrangement of the quasiparticle distribution may be regarded as a *precursor* of pion condensation. *Second*, the bifurcation point corresponding to formation of a hole bubble in the neutron momentum distribution is positioned at small momenta, $p_0 < 0.2p_F$, irrespective of the applicable value of $\rho_{c\pi}$. *Third*, the spectrum $\xi(p)$ shows a deep depression for $p \sim (0.5 - 0.6)p_F$. And *fourth*, the ratios $\rho_{cF}/\rho_{c\pi}$ and p_0/p_F are insensitive to the actual value taken by $\rho_{c\pi}$ within the usual range of theoretical predictions.

Analogous considerations apply to the proton subsystem of neutron-star matter, in which case one is dealing with the charged pion mode. Estimates ([11, 12]) indicate that this mode is also softened in dense matter, with a critical density not far from that for neutral pion condensation. One may then argue that, under the influence of the strongly momentum-dependent external field provided by the neutron medium, protons will leave the old Fermi sphere and occupy states of relatively large momentum, $p \sim 0.5p_{Fn}$. The impact of this further rearrangement on the proton-neutron ratio and on the rate of neutrino cooling requires a separate analysis, which we defer.

Having laid the microscopic basis for a rearrangement of the neutron Fermi surface that creates a bubble at low momenta in the Fermi sea, we return to its most striking astrophysical implication. Beyond the bifurcation point, the triangle inequalities can now be satisfied without the conventional requirement ([9, 2])) that the proton fraction exceed some 11–14%. Accordingly, the direct Urca process becomes active in the density regime just short of the threshold for pion condensation. At temperatures T above the anticipated superfluid phase transition, the T-dependence of the resulting neutrino emissivity is determined through the usual expression

$$\begin{aligned}\epsilon(n \to pe^-\bar{\nu}) \;&=\; \frac{2\pi}{\hbar} 2G_F^2(1+3g_A^2) \sum_i \varepsilon_\nu n_n (1-n_p)(1-n_e) \\ &\qquad \times \delta(\mathbf{p}_n - \mathbf{p}_p - \mathbf{p}_e - \mathbf{p}_\nu)\delta(\varepsilon_n - \varepsilon_p - \varepsilon_e - \varepsilon_\nu)\,, \end{aligned} \tag{6}$$

derived in quasiparticle approximation in terms of the occupations n_i and energies ε_i of the reacting particles. We do not consider a renormalization of the coupling constants G_F^2 and g_A^2 due to medium effects, and the emissivity takes the customary form ([9])

$$\epsilon(n \to pe^-\bar{\nu}) \simeq 1.2 \times 10^{27} \frac{M_n^*}{M_n} \frac{M_p^*}{M_p} \left(\frac{\mu_e}{100\ \mathrm{MeV}} \right) T_9^6\ \mathrm{erg\,cm^{-3}\,sec^{-1}}\,, \tag{7}$$

where μ_e is the electron chemical potential and the temperature T is measured in multiples of 10^9 K. Since the neutron and proton effective masses M_n^* and M_p^* remain T-independent, the bubble rearrangement serves only to turn on the direct Urca process at a lower density, without altering the familiar T^6 power-law behavior of the emissivity.

3. FERMION CONDENSATION

Interestingly enough, there exists a more radical scenario for rearrangement of the quasiparticle distribution, known as fermion condensation ([8, 14]). In this case, the occupancy $n(p)$ may be partial, i.e., it may lie between 0 and 1. At $T = 0$, the new quasiparticle distribution $n(p)$ is to be found from the variational condition

$$\frac{\delta E_0[n(p)]}{\delta n(p)} = \mu, \qquad p \in \Omega\,. \tag{8}$$

The left-hand side of equation (8) is just the quasiparticle energy $\varepsilon(p)$. Hence its coincidence with the chemical potential μ means that the spectrum of single-particle excitations will be dispersionless (i.e., the quasiparticle group velocity will vanish) throughout the entire momentum domain Ω where $\xi(p) = \varepsilon(p) - \mu = 0$, and not merely at isolated points as implied by equation (2). The family of quasiparticles having momenta $p \in \Omega$ is called the fermion condensate because of a conspicuous analogy with the low-temperature Bose gas, in which the energy of condensate particles is also equal to the chemical potential μ. A key signature of fermion condensation has been observed in strongly correlated electron systems ([13]): flat portions of the single-particle spectrum have been seen experimentally in a number of high-temperature superconductors ([17]).

The two types of rearrangement – bubble formation and fermion condensation – can compete with each other in the density regime just below the soft-mode phase transition. Numerical studies ([19]) demonstrate that fermion condensation wins the contest at nonzero T. Let us suppose this is the case in the neutron-star medium, while continuing to disregard nucleonic pairing phenomena. Consistency of the Fermi-Dirac form $n(p,T) = \{\exp[\xi(p;n(p,T))/T] + 1\}^{-1}$ for the quasiparticle distribution with the variational condition (8) requires that the spectrum $\xi(p,T)$ of the fermion-condensate phase grows linearly with T at low temperature, implying an effective mass inversely proportional to T ([8, 14]).

Although the rearranged momentum distribution derived from equation (8) differs from the bubble configuration, its structure will also admit thermal excitations at low neutron momenta. Hence we may again expect most neutron stars to contain a region of relatively moderate density, bounded below by ρ_{cF} and above by $\rho_{c\pi}$, in which the direct Urca process operates vigorously. However, due to the new feature of a T-dependent neutron effective mass, $M_n \propto 1/T$, we may anticipate an enhancement of the neutrino emissivity relative to the standard result ([9]), corresponding to a T^5 rather than a T^6 dependence on the temperature.

4. CONCLUSIONS

We have explored the possibility that an effective interaction with strong momentum dependence gives rise to a rearrangement of the neutron momentum distribution in neutron-star matter. Two plausible manifestations of this phase transformation – creation of a doubly-connected Fermi surface and fermion condensation – have been considered. Both open the prospect that direct nucleon Urca cooling is present in a density regime just below the threshold for pion condensation and consequently at a density much lower than previously estimated. If a fermion condensate is formed, the resulting neutrino emissivity is significantly larger than that generated by the direct Urca process in normal matter. Within the affected density range, it would therefore dominate all other proposed neutrino cooling mechanisms ([15]). Future studies along this line will focus on temperatures below the superfluid transition and on the effect of the dramatically increased emissivity on neutrino opacity.

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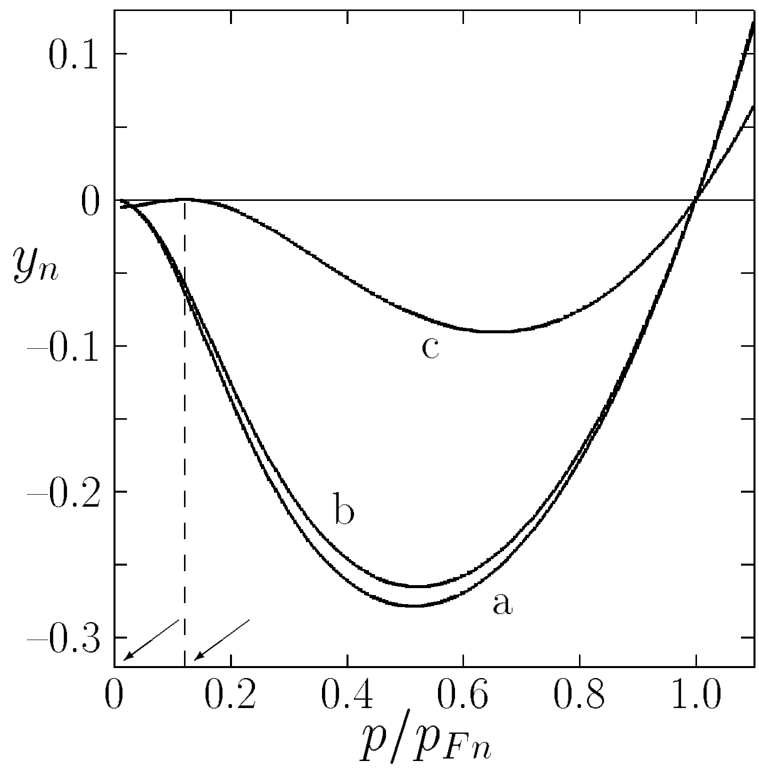
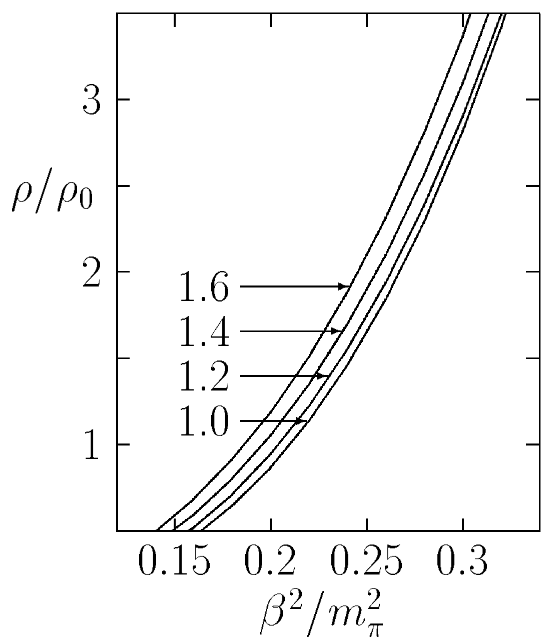
FIGURE CAPTIONS

Fig. 1. The dimensionless neutron spectrum $y_n(p) = \xi_n(p)/(p_F^2/2M)$ at the critical densities ρ_{cF} corresponding to three different sets of model parameters: (a) $\gamma = 1.25m_\pi$, $k_0 = 0.9\, p_{Fn}$, $\beta^2 = 0.22\, m_\pi^2$ ($\rho_{cF} \simeq 1.19\, \rho_0$), (b) $\gamma = 1.25m_\pi$, $k_0 = 0.9\, p_{Fn}$, $\beta^2 = 0.25\, m_\pi^2$ ($\rho_{cF} \simeq 1.76\, \rho_0$), (c) $\gamma = 1.25m_\pi$, $k_0 = p_{Fn}$, $\beta^2 = 0.13\, m_\pi^2$ ($\rho_{cF} \simeq 1.88\, \rho_0$). Two different positions of the bifurcation point, namely $p_0 = 0$ (for parameter sets (a) and (b)) and $p_0 \simeq 0.12\, p_{Fn}$ (for set (c)), are indicated by arrows.

Fig. 2. Phase diagram of neutron matter in the variables ρ (measured in ρ_0) and β^2 (measured in m_π^2), as calculated for $k_0 = 0.9\, p_{Fn}$ and four different values of γ, which (in m_π units) label the corresponding the phase boundaries separating the bubble phase (upper left) from the normal phase (lower right).



0.1
0
y_n
−0.1
−0.2
−0.3
0
0.2
0.4
0.6
0.8
1.0
p/p_{Fn}
a
b
c



3
2
1
ρ/ρ_0
1.6
1.4
1.2
1.0
0.15
0.2
0.25
0.3
β^2/m_π^2